

Frame

A pin-connected frame differs from a truss in that the members may no longer be connected at their ends and the loads may be applied at any point to the structure.

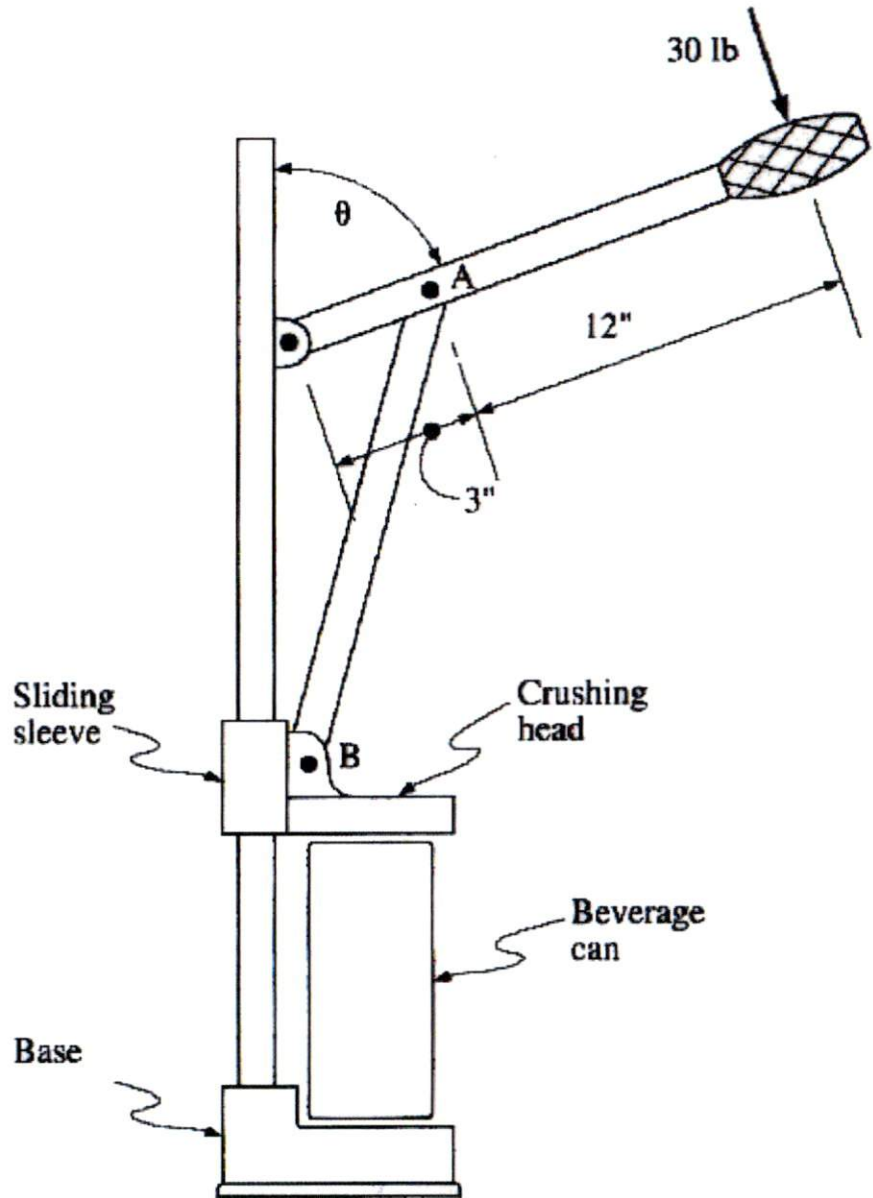
At least one of the members must be acted on by three or more forces. Thus the forces at the joints that hold the frame together must have x and y components.

Machine

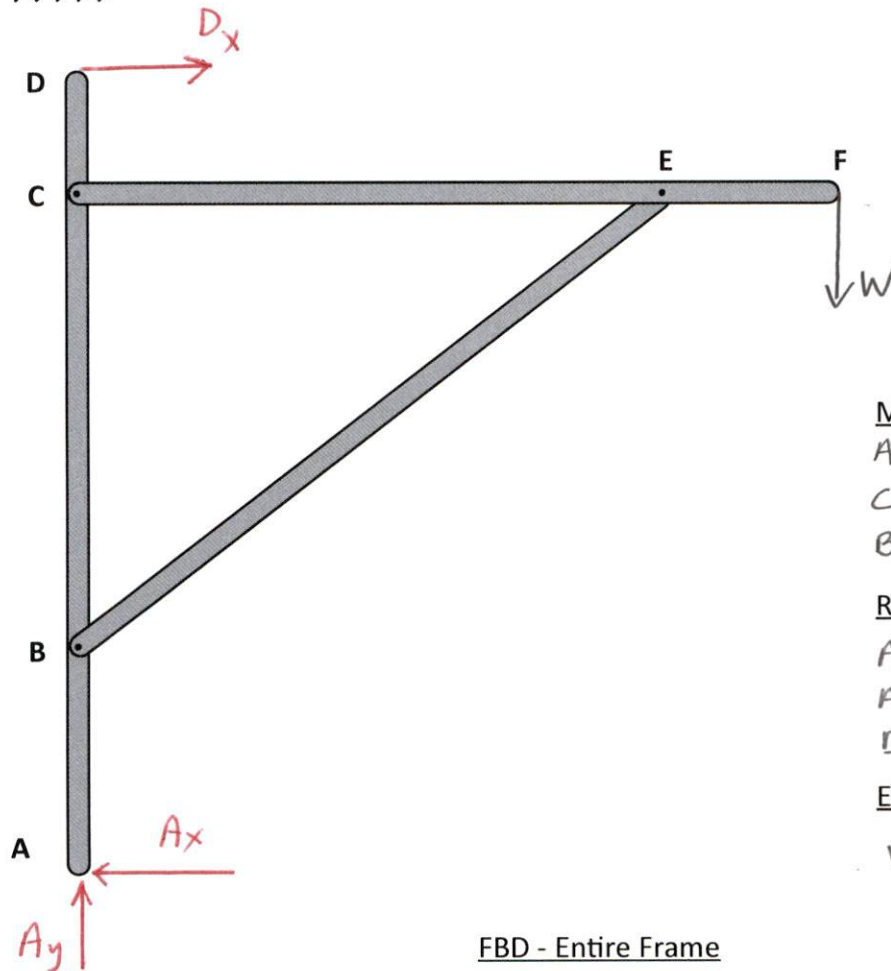
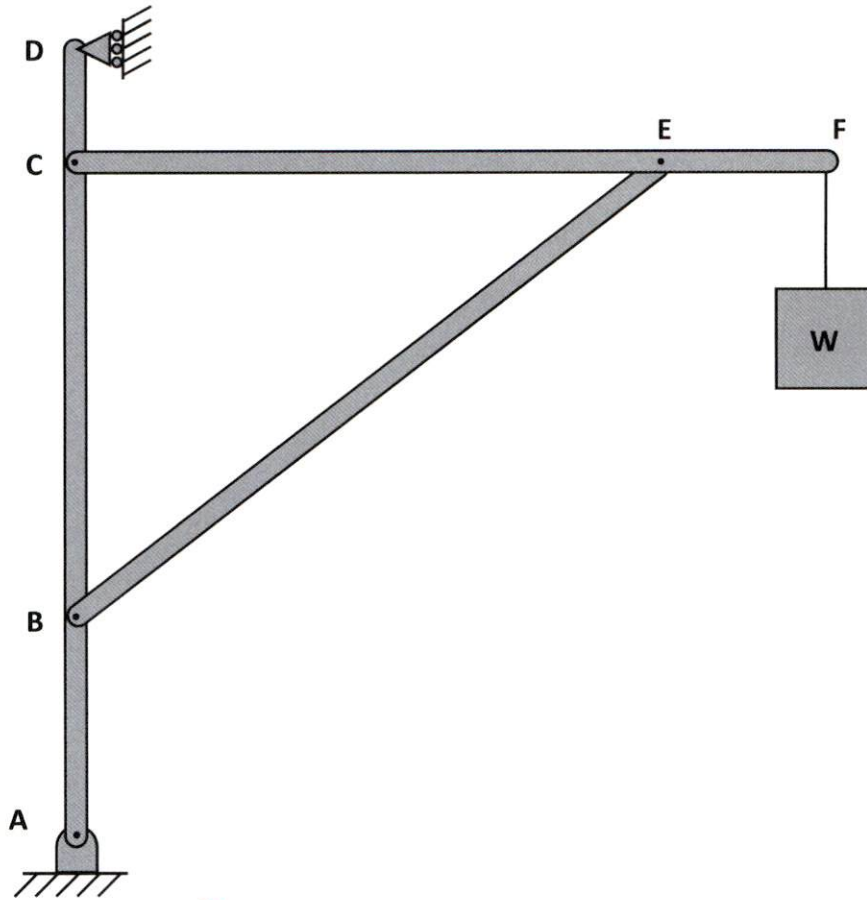
A machine differs from a pin-connected frame in that it is designed to transmit and transform input forces into output forces rather than support loads.

Frames are rigid structures while machines are nonrigid structures.

In machines, we will be concerned with the relationship between input and output forces necessary for equilibrium.



Frame



Members

ABCD

CEF

BE (TFM)

Reactions

A_x

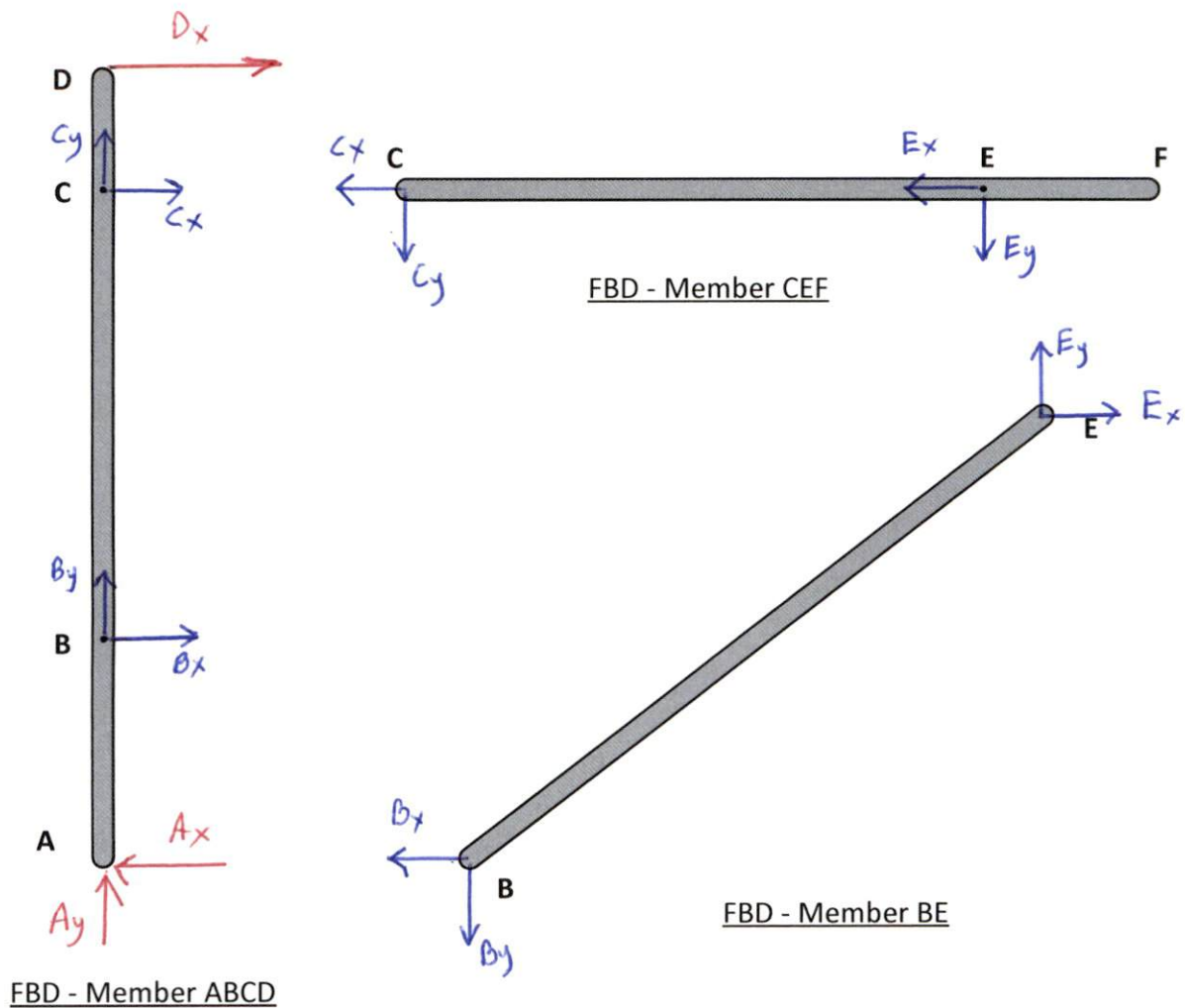
A_y

D_x

External Load

W

FBD - Entire Frame



Free Body Diagrams for the three members of the frame

Newton's Third Law – action and reaction

Must be observed when forces are drawn on the FBD's of the connected members.

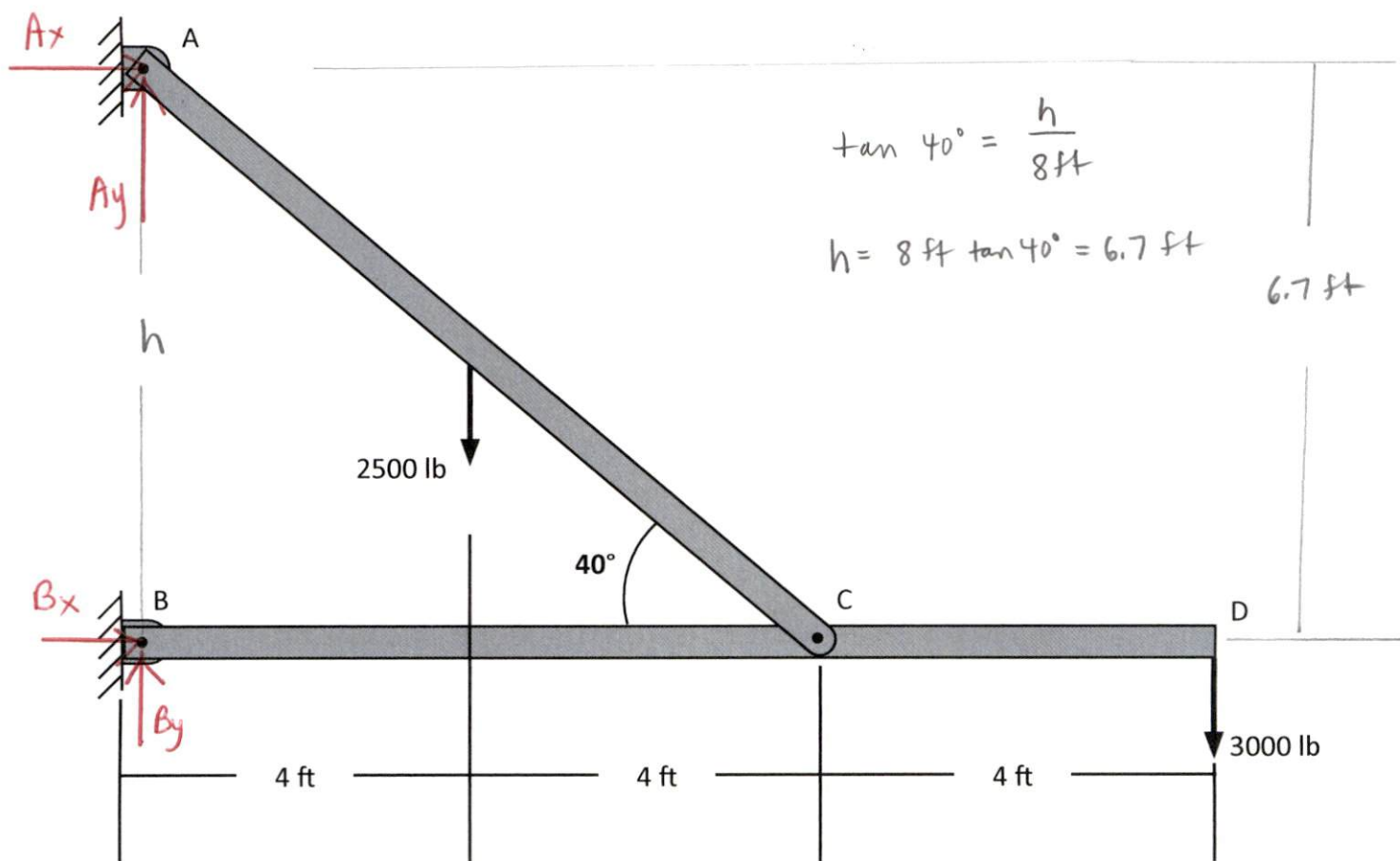
Member BE is a Two-Forec Member (TFM) and the force in the member acts along the member.

How do you check you correctly drew the forces on each FBD?

If the three free-body diagrams for the parts of the frame are combined, the forces at B, C, and E cancel and we have a FBD for the entire frame.

Example 1

A bracket is pin connected at points A, B, and D and is subjected to the loads shown. Calculate the pin reactions. Neglect the weights of the members.



Solution.

FBD - Entire Frame

ccw + M ↺
cw - M ↻

Equilibrium Equations

$$(\sum M_A = 0) \quad B_x (6.7 \text{ ft}) - 2500 \text{ lb} (4 \text{ ft}) - 3000 \text{ lb} (12 \text{ ft}) = 0$$

$$B_x = \frac{46,000 \text{ lb} \cdot \text{ft}}{6.7 \text{ ft}} = \underline{\underline{6866 \text{ lb} \rightarrow}}$$

$$(\sum F_x = 0) \quad A_x + B_x = 0$$

$$A_x = -B_x = -6866 \text{ lb} \rightarrow$$

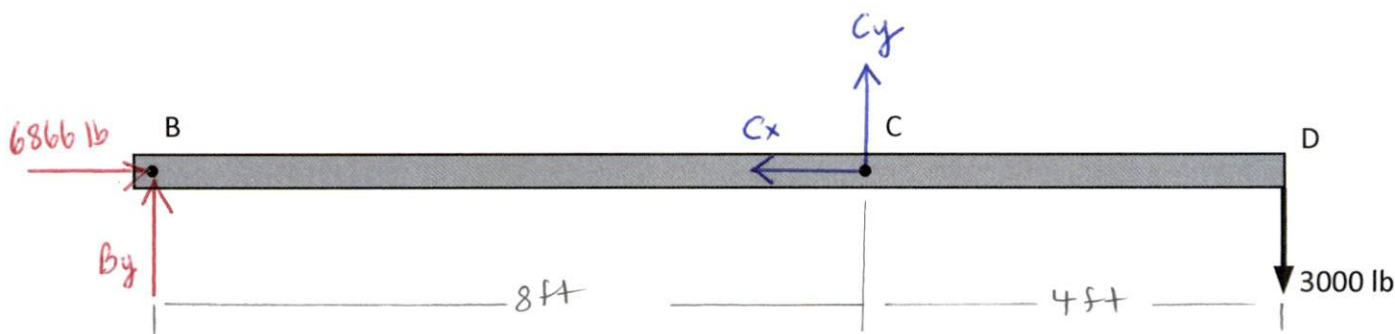
and $A_x = 6866 \text{ lb} \leftarrow$

$$(\sum F_y = 0) \quad A_y + B_y - 2500 \text{ lb} - 3000 \text{ lb} = 0$$

$$A_y + B_y = 5500 \text{ lb} \quad \left(\begin{array}{l} \text{can't solve} \\ \text{yet} \end{array} \right)$$

EQN (1)

Member BCD



FBD - Member BCD

ccw + M ↺
cw - M ↻

Equilibrium Equations

$$[\sum M_c = 0] \quad - B_y (8 \text{ ft}) - 3000 \text{ lb} (4 \text{ ft}) = 0$$

$$B_y = \frac{-12,000 \text{ lb} \cdot \text{ft}}{8 \text{ ft}} = -1500 \text{ lb} \uparrow$$

and $B_y = 1500 \text{ lb} \downarrow$

$$[\sum F_x = 0] \quad 6866 \text{ lb} - C_x = 0$$

$$C_x = \underline{\underline{6866 \text{ lb} \leftarrow}}$$

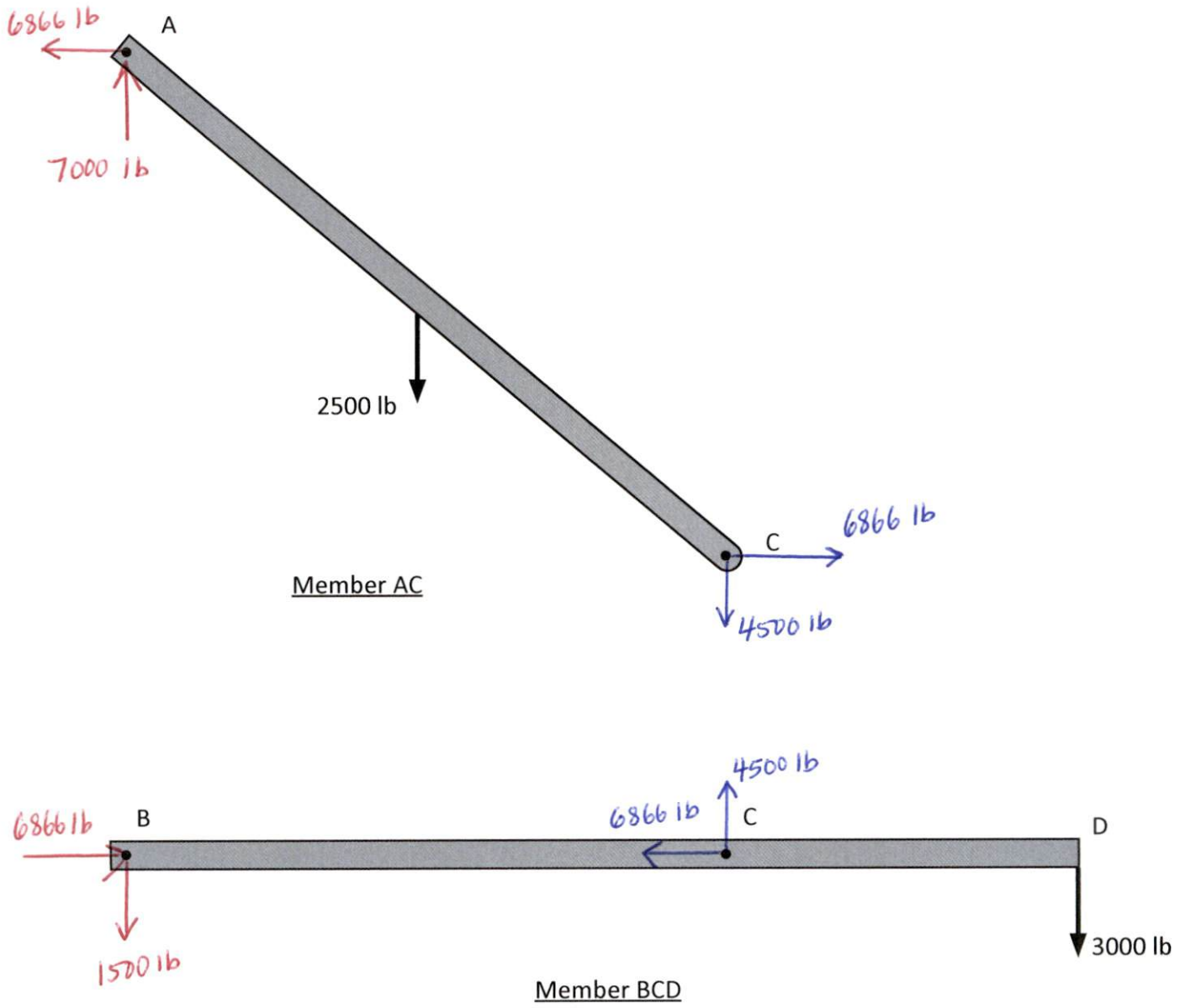
$$[\sum F_y = 0] \quad B_y + C_y - 3000 \text{ lb} = 0$$

$$C_y = 3000 \text{ lb} - (-1500 \text{ lb}) = \underline{\underline{4500 \text{ lb} \uparrow}}$$

From Egn (1) $A_y + B_y = 5500 \text{ lb}$

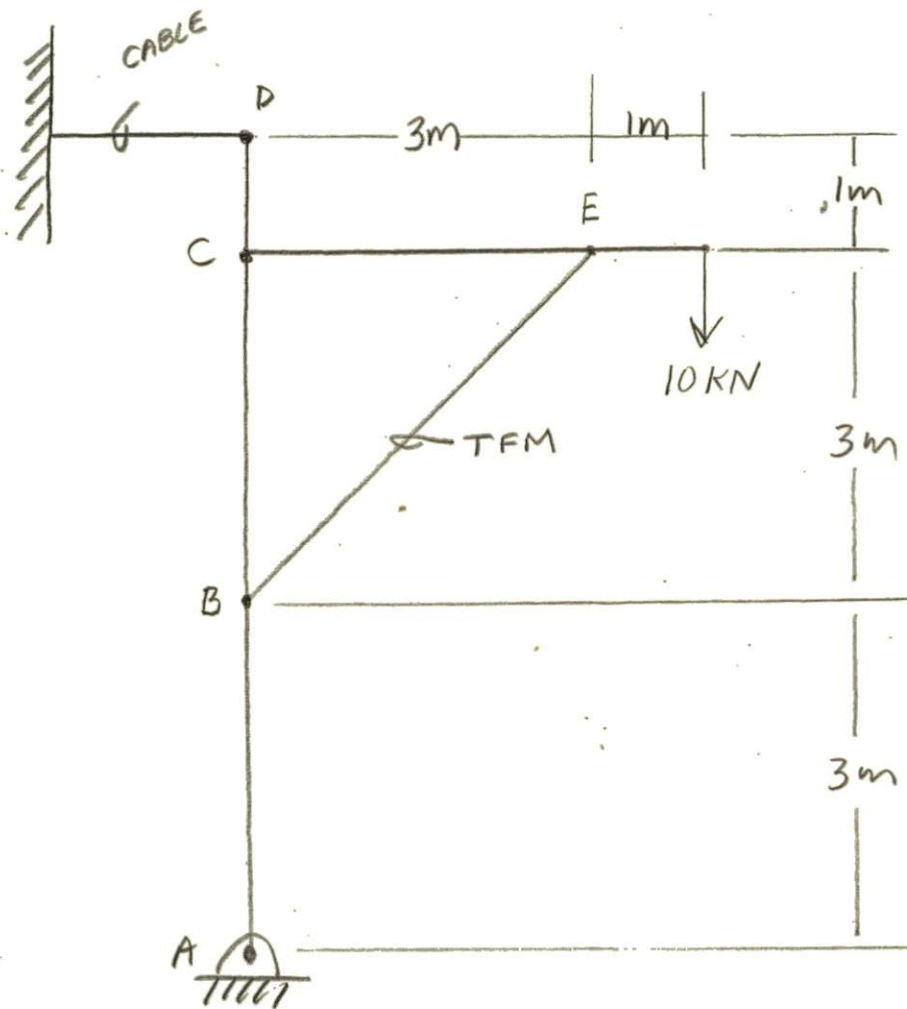
$$A_y = 5500 \text{ lb} - (-1500 \text{ lb}) = 7000 \text{ lb} \uparrow$$

Summarize the Results



Example 2. (solution #1)

Determine the pin reactions for the frame shown.



Solution.

MULTI-FORCE MEMBERS

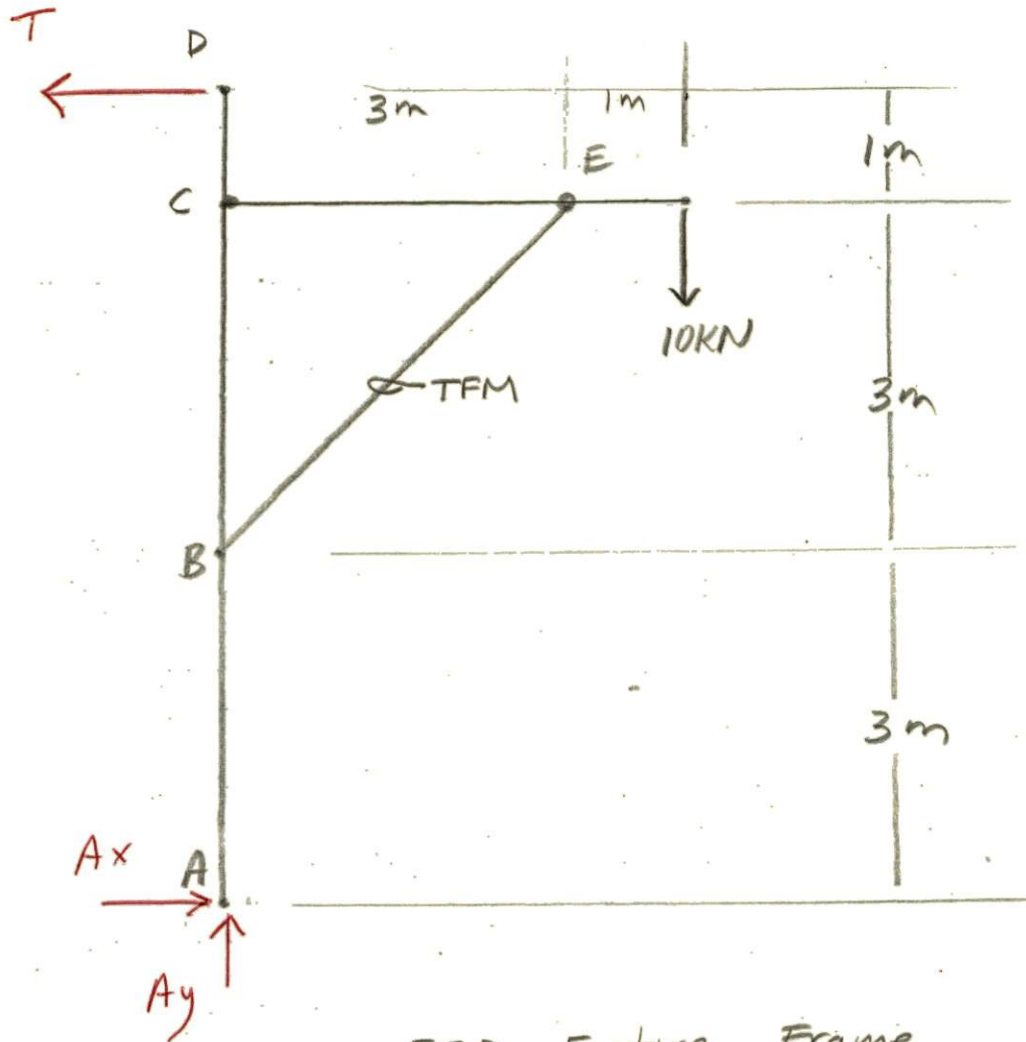
ABCD

CE

TFM

BE

Step 1. Use the FBD of the Entire Frame to solve for the support reactions.



FBD - Entire Frame

Equilibrium Equations

ccw + M ↺
cw - M ↻

$$[\sum M_A = 0] \quad T(7m) - 10kN(4m) = 0$$

$$T = \frac{40 \text{ kN} \cdot m}{7m}$$

$$T = \underline{\underline{5.71 \text{ kN}}} \leftarrow$$

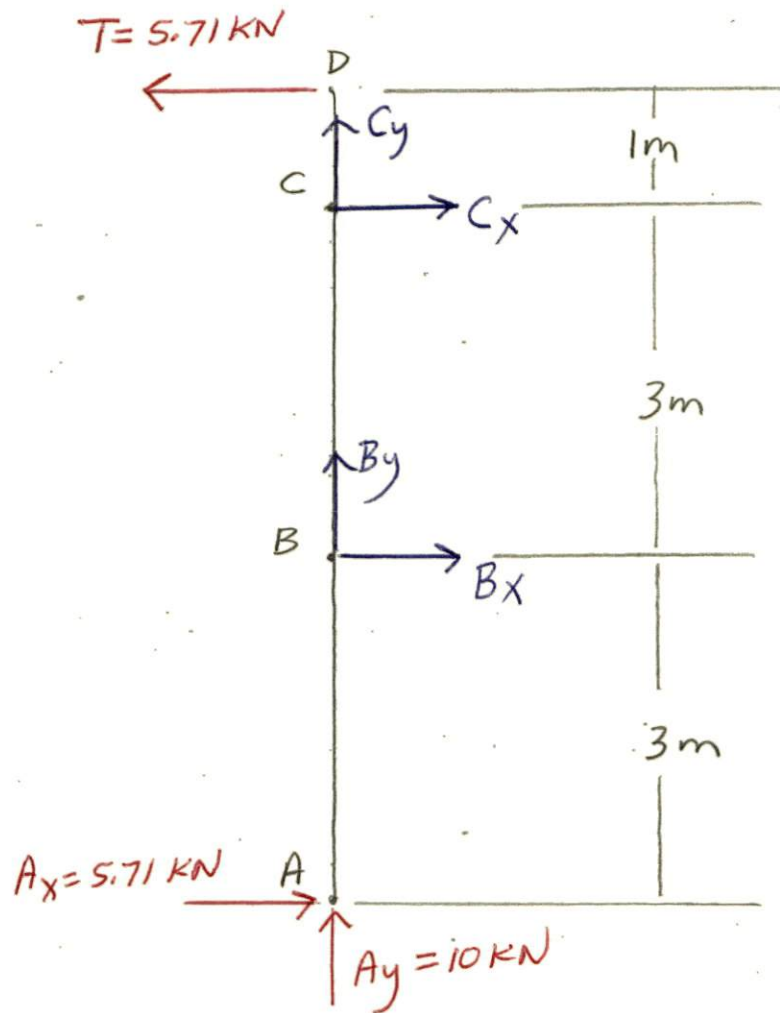
$$[\sum F_x = 0] \quad A_x - T = 0$$

$$A_x = \underline{\underline{5.71 \text{ kN}}} \rightarrow$$

$$[\sum F_y = 0] \quad A_y - 10 \text{ kN} = 0$$

$$A_y = \underline{\underline{10 \text{ kN} \uparrow}}$$

Member ABCD



FBD - member ABCD

Equilibrium Equations

$$[\sum M_B = 0] \quad 5.71 \text{ kN}(4\text{m}) + 5.71 \text{ kN}(3\text{m}) - C_x(3\text{m}) = 0$$

$$C_x = \frac{39.97 \text{ kN}\cdot\text{m}}{3\text{m}} = \underline{\underline{13.3 \text{ kN} \rightarrow}}$$

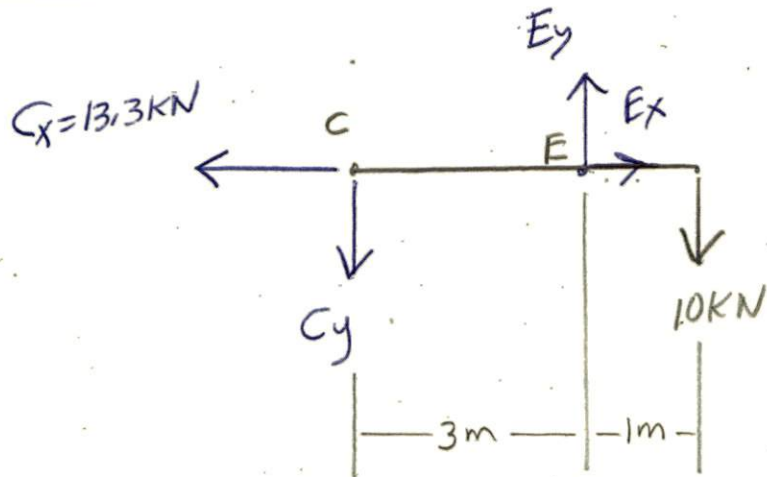
$$[\sum F_x = 0] \quad 5.71 \text{ kN} - 5.71 \text{ kN} + C_x + B_x = 0$$

$$B_x = -13.3 \text{ kN} \rightarrow$$

$$\text{or } \boxed{B_x = 13.3 \text{ kN} \leftarrow}$$

$$[\sum F_y = 0] \quad C_y + B_y + 10 \text{ kN} = 0 \quad (\text{Eqn 1}) \quad (\text{C.S.Y})$$

Member CE



FBD - member CE

Equilibrium Equations

$$[\sum M_c = 0] \quad E_y(3\text{m}) - 10 \text{ kN}(4\text{m}) = 0$$

$$E_y = \underline{\underline{13.3 \text{ kN} \uparrow}}$$

$$[\sum F_x = 0] \quad -13.3 \text{ kN} + E_x = 0$$

$$E_x = \underline{\underline{13.3 \text{ kN} \rightarrow}}$$

$$[\sum F_y = 0] \quad -C_y + E_y - 10 \text{ kN} = 0$$

$$C_y = 13.3 \text{ kN} - 10 \text{ kN}$$

$$C_y = \underline{\underline{3.3 \text{ kN} \downarrow}}$$

From (Eqn.1)

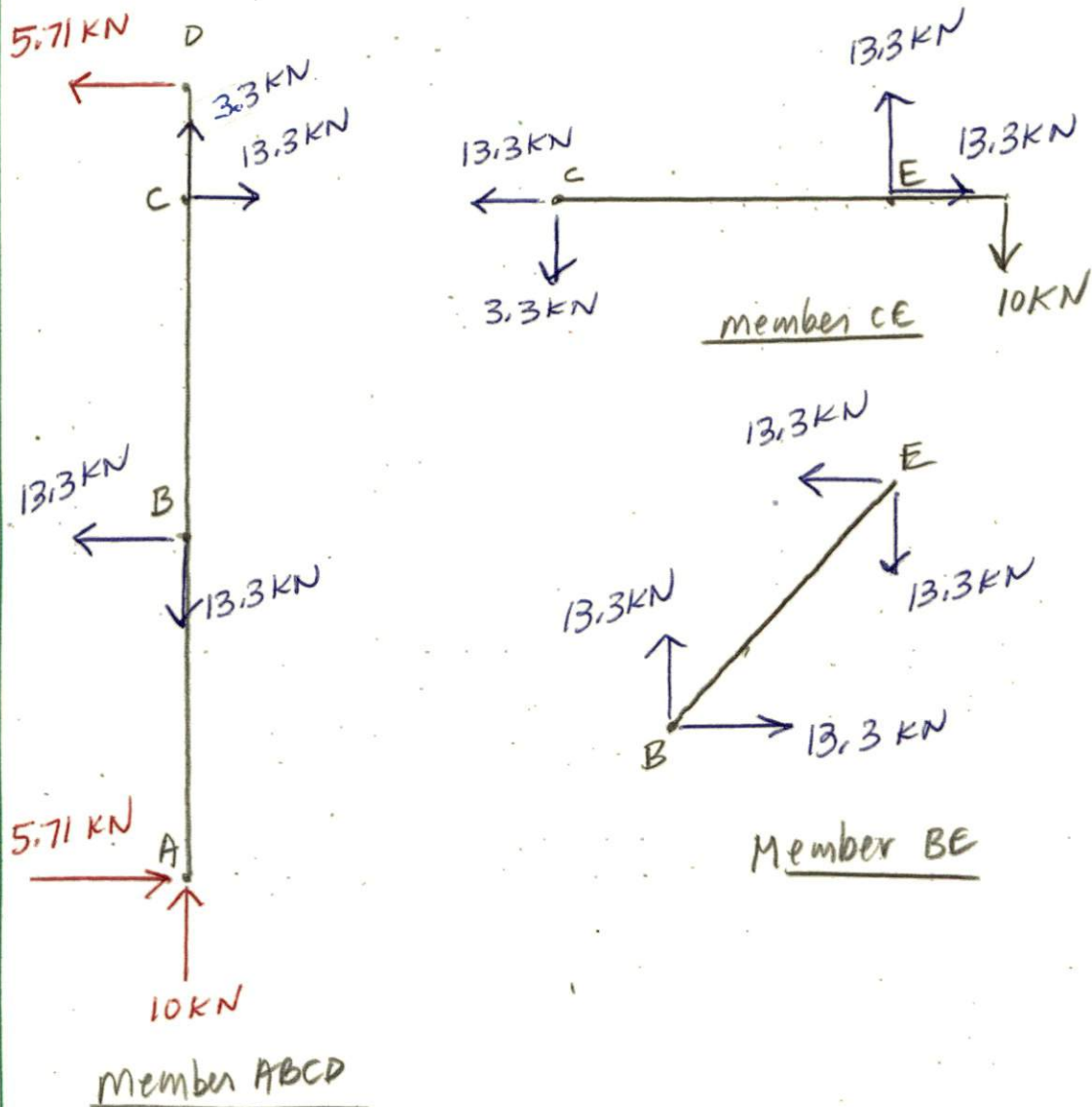
$$C_y + B_y + 10 \text{ kN} = 0$$

$$B_y = -3.3 \text{ kN} - 10 \text{ kN}$$

$$B_y = -13.3 \text{ kN} \uparrow$$

or $B_y = 13.3 \text{ kN} \downarrow$

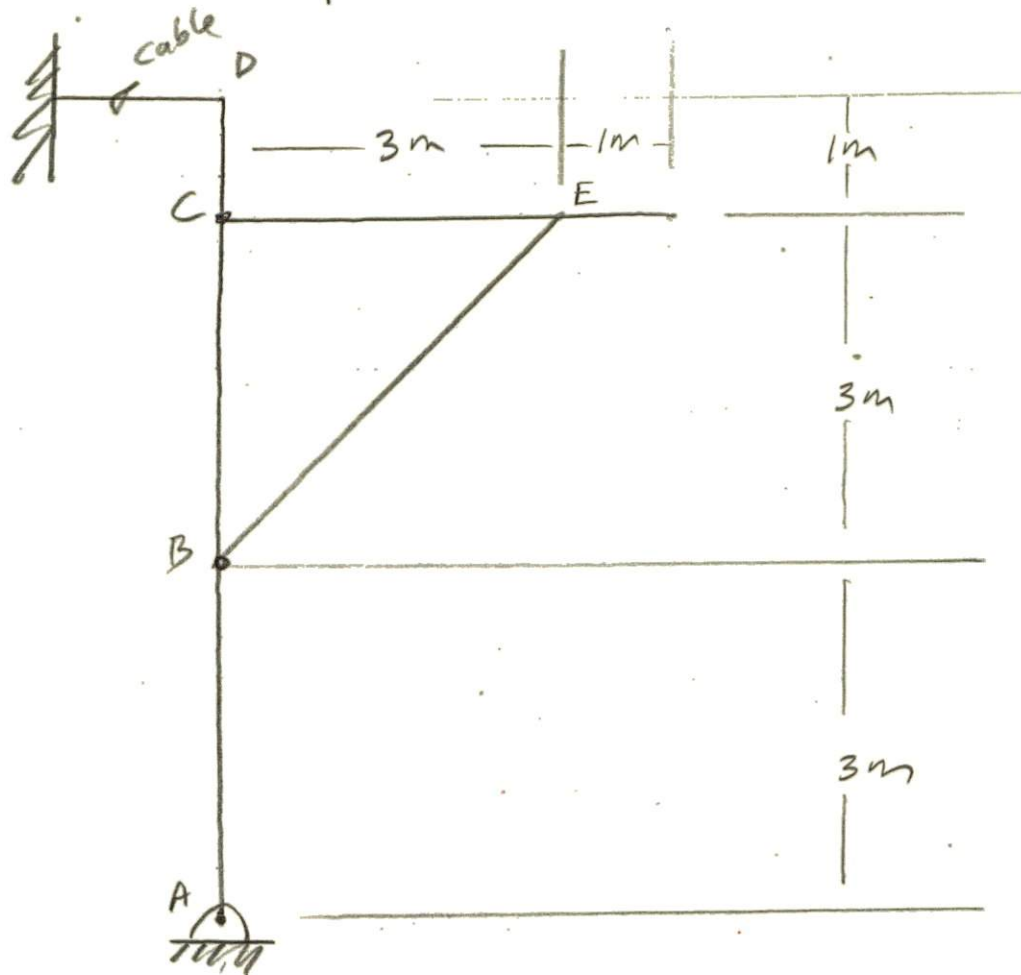
Summarize Results (No Dimensions shown)



All members are in Equilibrium!

Example 2. (solution #2)

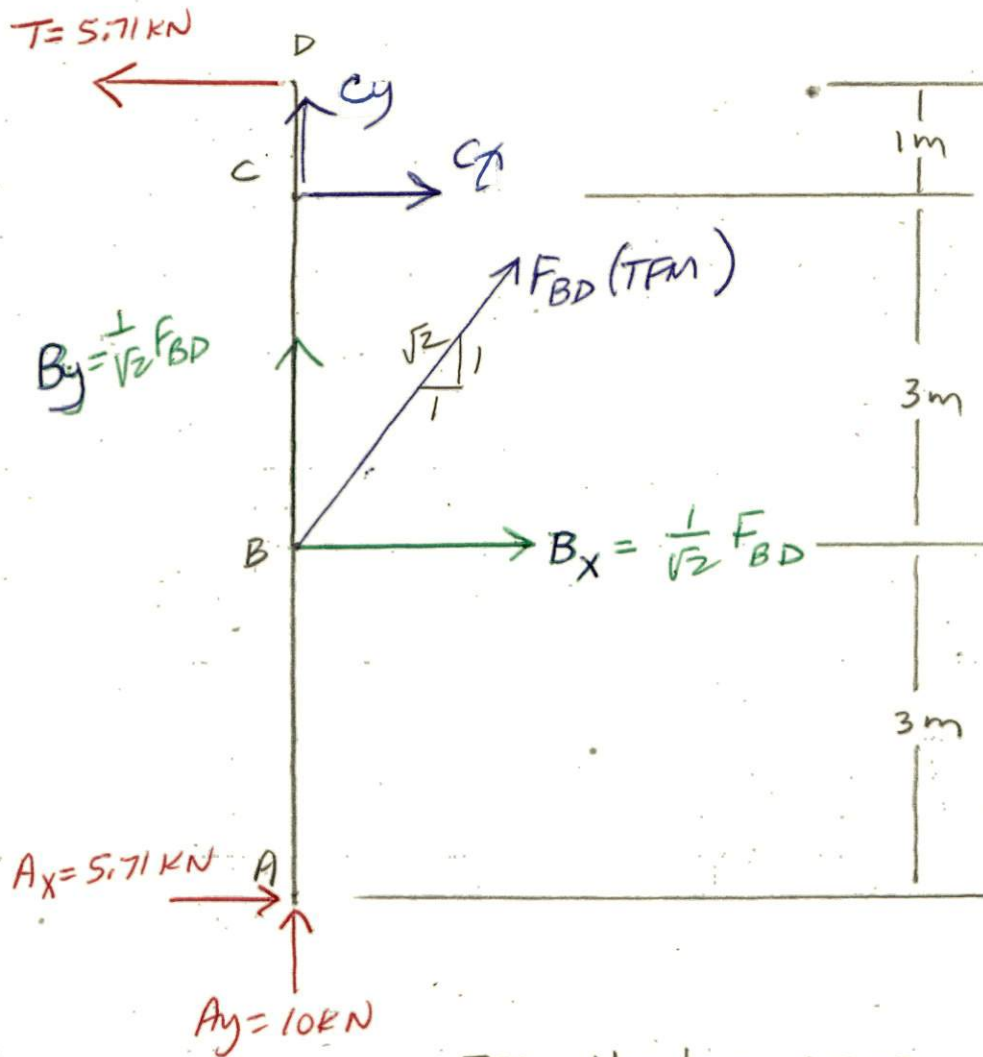
Determine the pin reactions for the frame shown.



Solution.

See solution #1 to solve for the reactions at supports A and D.

Member ABCD



FBD - Member ABCD

Equilibrium Equations

$$[\sum M_B = 0] \quad 5.71 \text{ kN}(4\text{m}) + 5.71 \text{ kN}(3\text{m}) - C_x(3\text{m}) = 0$$

$$C_x = 13.3 \text{ kN} \rightarrow$$

$$[\sum F_x = 0] \quad -5.71 \text{ kN} + 5.71 \text{ kN} + C_x + \frac{1}{\sqrt{2}} F_{BD} = 0$$

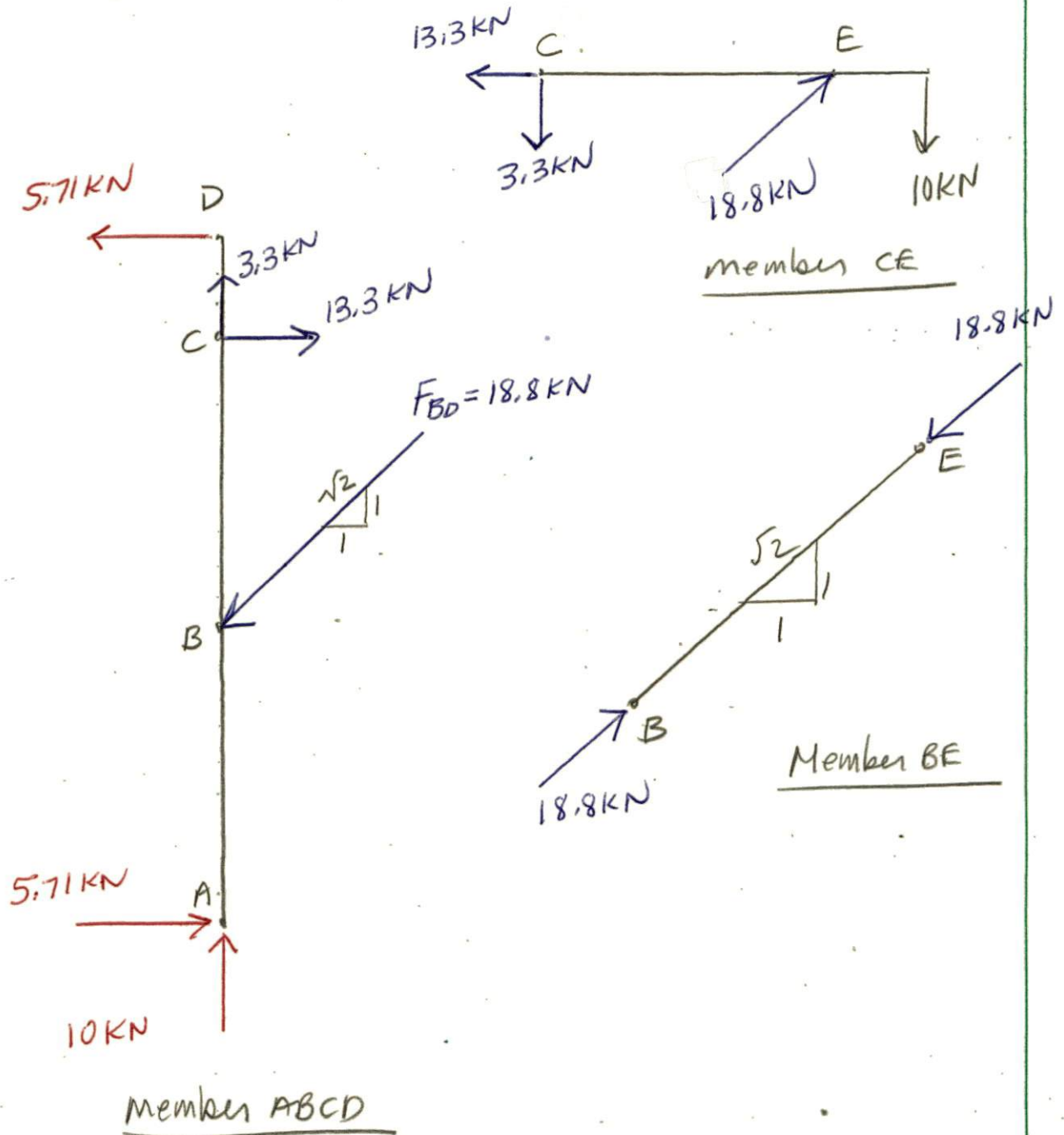
$$F_{BD} = -13.3 \text{ kN}(\sqrt{2}) = -18.8 \text{ kN}(T)$$

$$[\sum F_y = 0] \quad 10 \text{ kN} + \frac{1}{\sqrt{2}} F_{BD} + C_y = 0$$

$$C_y = -10 \text{ kN} - \frac{1}{\sqrt{2}}(-18.8 \text{ kN}) = \underline{\underline{3.3 \text{ kN} \uparrow}}$$

or $F_{BD} = 18.8 \text{ kN}(C)$

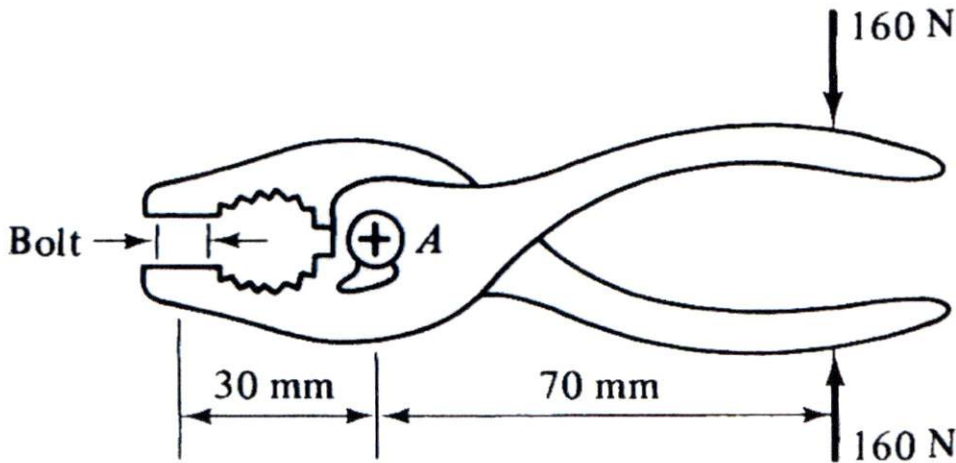
Summarize Results



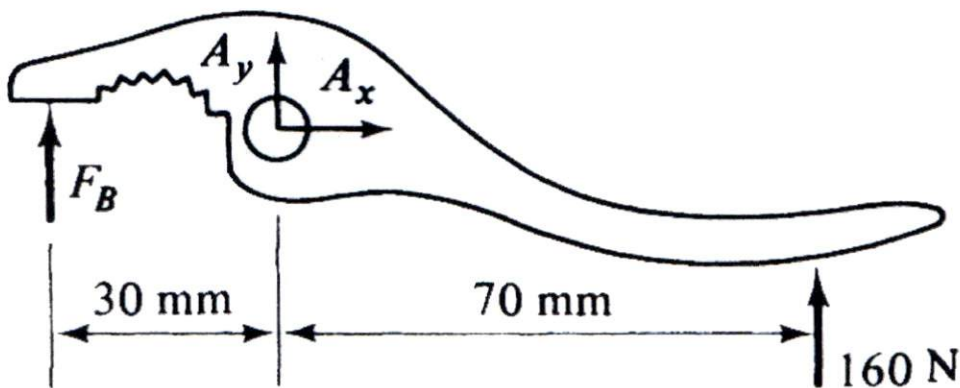
Machines

Example 1

A force is applied to the handle of the pliers shown. Find the force applied to the bolt and the horizontal and vertical reactions at the hinge A.



Solution.



Free-Body Diagram - Upper handle of Pliers

Equilibrium Equations

ccw + M ↺
cw - M ↻

$$[\sum M_A = 0] \quad -F_B (30\text{mm}) + 160\text{N} (70\text{mm}) = 0$$

$$F_B = \underline{\underline{373\text{N}}}$$

$$[\sum F_x = 0] \quad A_x = 0$$

$$[\sum F_y = 0] \quad F_B + A_y + 160\text{N} = 0$$

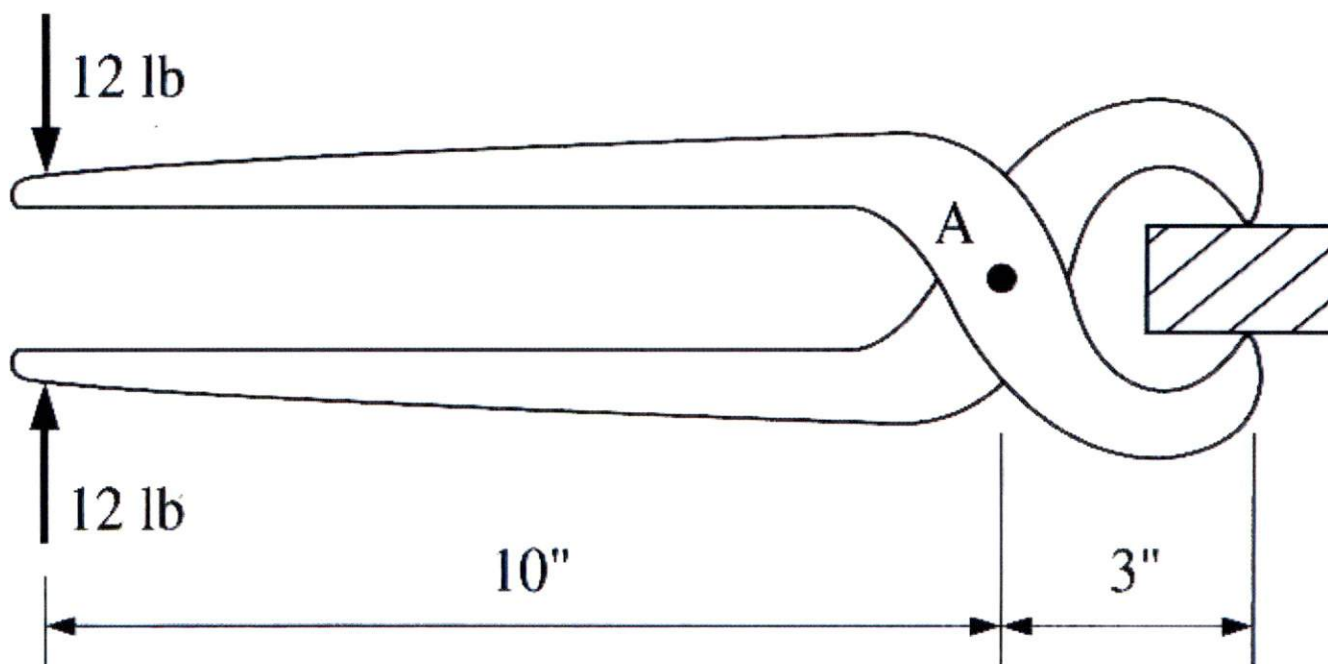
$$A_y = -373\text{N} - 160\text{N} = -533\text{N} \uparrow$$

and

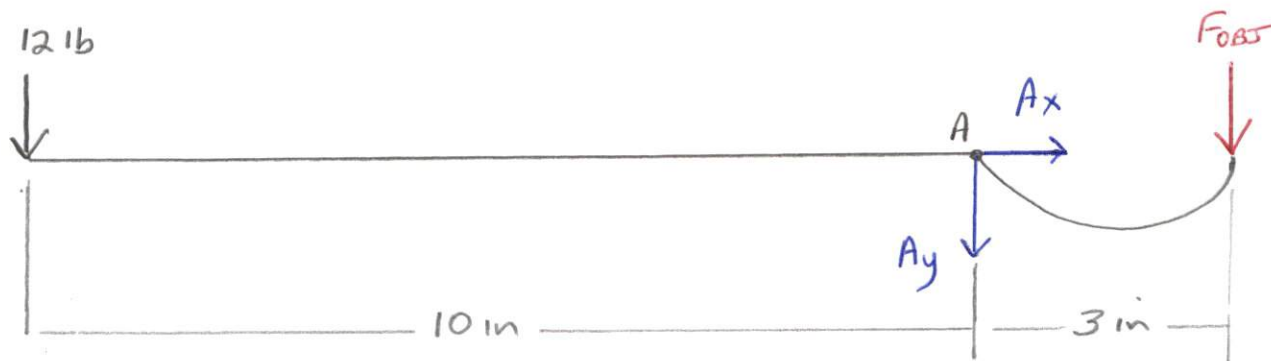
$$A_y = 533\text{N} \downarrow$$

Example 2

The tongs shown are used to grip an object. For an input force of 12 lb on each handle, determine the forces exerted on the object and the forces exerted on the pin at A.



Solution.



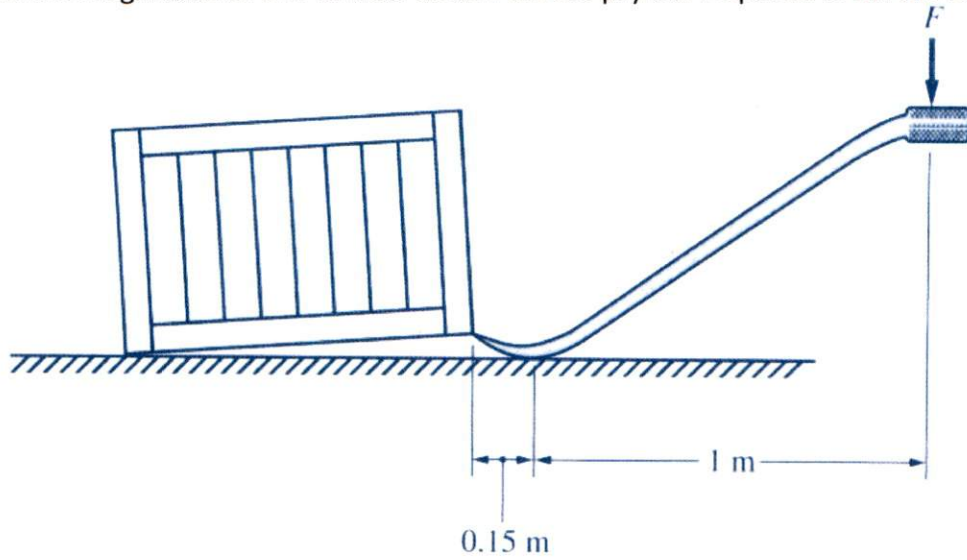
FBD - Upper member

Equilibrium Equations

$$[\sum M_A = 0] \quad 12 \text{ lb} (10 \text{ in}) - F_{\text{obj}} (3 \text{ in}) = 0$$

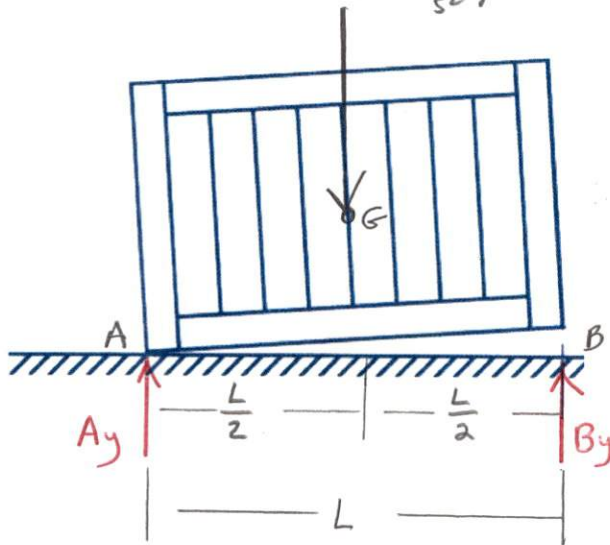
$$F_{\text{obj}} = \frac{120 \text{ lb} \cdot \text{in}}{3 \text{ in}} = \underline{\underline{40 \text{ lb} \downarrow}}$$

4-39 Determine the magnitude of the vertical force F on the pry bar required to lift the 2000-kg crate.



Solution.

$$W = 2000 \text{ kg} \left(9.81 \frac{\text{m}}{\text{s}^2} \right) = 19.62 \text{ kN}$$



FBD - Crate

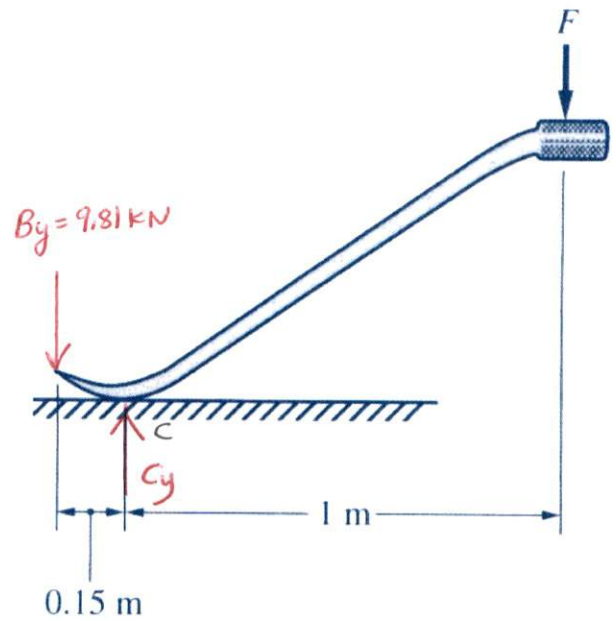
Equilibrium Equations

$$[\sum M_A = 0]$$

$$-19.62 \text{ kN} \left(\frac{L}{2} \right) + B_y(L) = 0$$

$$B_y = \frac{19.62 \text{ kN} \left(\frac{L}{2} \right)}{L}$$

$$= \underline{\underline{9.81 \text{ kN} \uparrow}}$$



FBD - Pry Bar

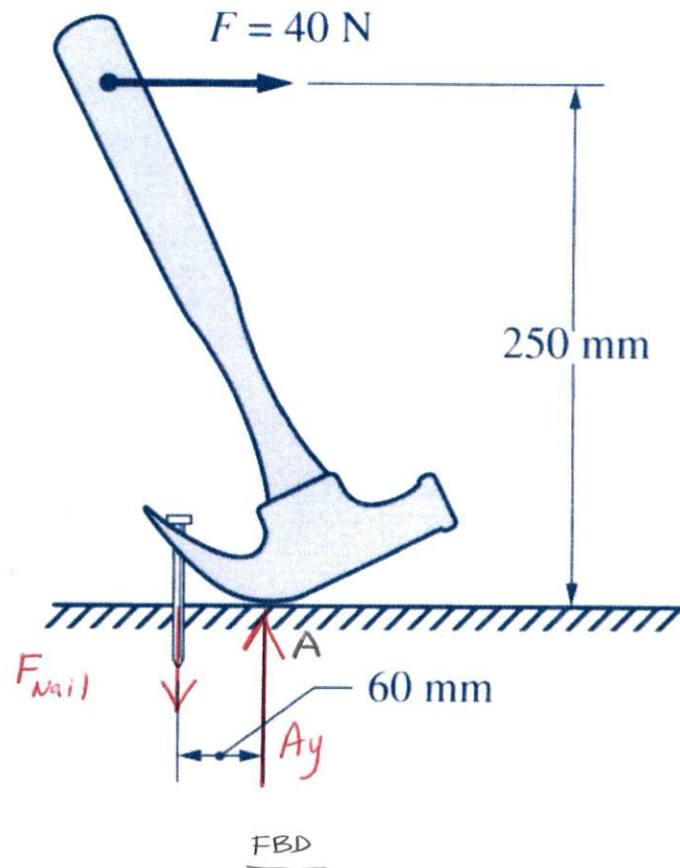
Equilibrium Equations

$$[\sum M_c = 0]$$

$$9.81 \text{ kN} (0.15 \text{ m}) - F(1 \text{ m}) = 0$$

$$F = \underline{\underline{1.47 \text{ kN} \downarrow}}$$

4-40 A horizontal force F of 40N is applied to the claw hammer. Determine the force exerted on the nail by the claw hammer.



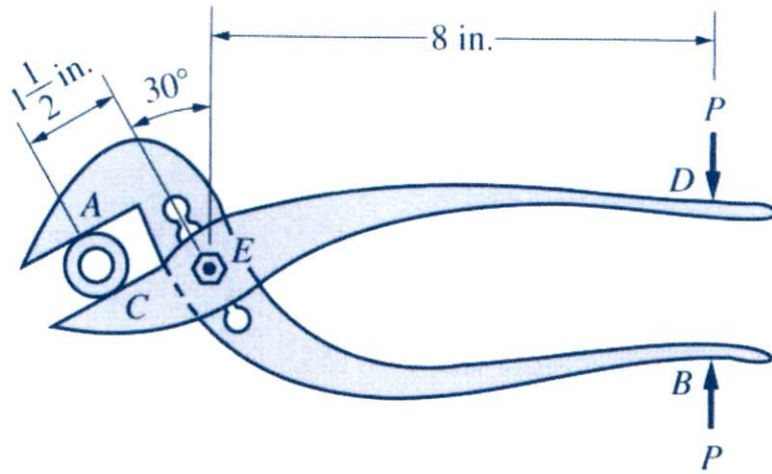
Solution.

Equilibrium Equations

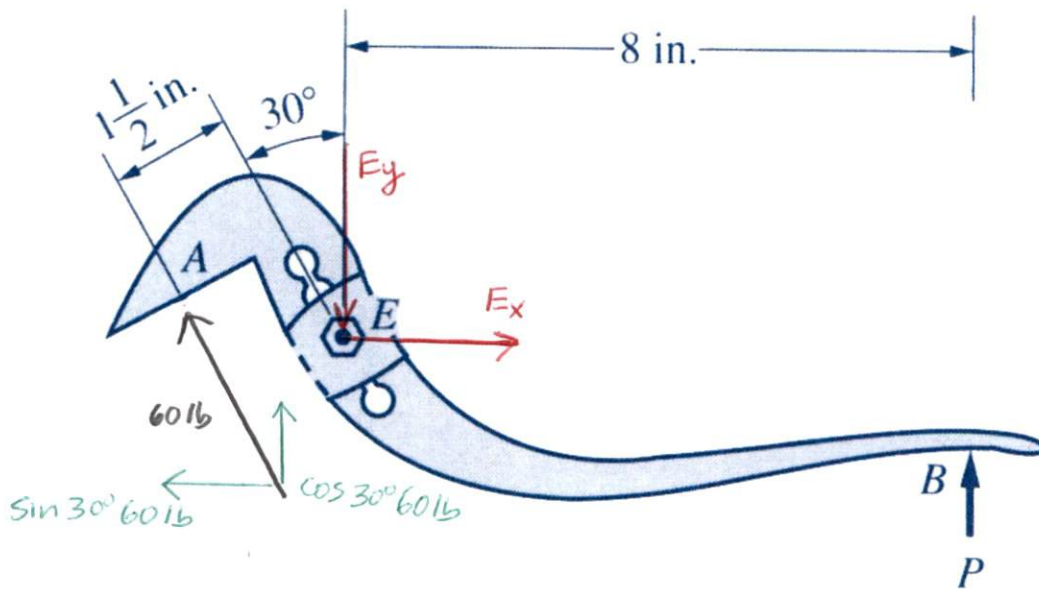
$$[\sum M_A = 0] \quad F_{\text{NAIL}} (60\text{ mm}) - 40\text{ N} (250\text{ mm}) = 0$$

$$F_{\text{Nail}} = \frac{40\text{ N} (250\text{ mm})}{60\text{ mm}} = \underline{\underline{167\text{ N}}} \downarrow$$

4-41 The pipe is held by a joint plier with a clamping force of 60 lb. Determine (a) the force P applied to the handles and (b) the force exerted by the pin E on portion AB of the plier.



Solution.



Equilibrium Eqs

FBD- member AEB

(a)

$$[\sum M_E = 0] \quad P(8 \text{ in}) - 60 \text{ lb}(1.5 \text{ in}) = 0$$

$$P = \underline{\underline{11.25 \text{ lb} \uparrow}}$$

(b)

$$[\sum F_x = 0] \quad -\sin 30^\circ 60 \text{ lb} + E_x = 0$$

$$E_x = \underline{\underline{30 \text{ lb} \rightarrow}}$$

$$[\sum F_y = 0] \quad \cos 30^\circ 60 \text{ lb} - E_y + 11.25 \text{ lb} = 0$$

$$E_y = \underline{\underline{62.3 \text{ lb} \downarrow}}$$